PG – 221

II Semester M.Sc. Examination, June 2016
(CBCS)
MATHEMATICS
M 208 SC : Mathematical Modelling and Numerical Analysis – I

Time : 3 Hours   Max. Marks : 70

Instructions : *i) Answer **any five full** questions.*

*ii) **All** questions carry **equal** marks.*

1. a) Explain briefly the significance of mathematical modelling of a system and outline the important steps involved there in.

   b) Develop a mathematical model for the study of detection of diabetes through glucose tolerance test. **(7+7)**

2. a) Discuss the mathematical model to understand the concept of prey-predator system.

   b) Derive the one-dimensional wave equation for the vibration of a string without damping. **(7+7)**

3. a) If $\phi(x)$ is a continuous function in [a, b] containing the root and $|\phi'(x)| < 1$ in this interval then show that for any choice of $x_0 \in [a,b]$ the sequence $\{x_k\}$ determined from $x_{K+1} = \phi(x_K)$ K = 0, 1, 2, ... converges to the root ξ of $x = \phi(x)$.

   b) State Descarte's rule of signs and Strum's theorem. Using Strum sequence, find the number of real and complex roots of $4x^4 + 2x^2 - 1 = 0$. **(6+8)**

4. a) Define rate of convergence. Show that the Newton-Raphson method converges linearly for finding a multiple root of f(x) = 0.

   b) Using Ramanujan's method, obtain first six convergents to find a smallest real root of the equation $xe^x = 1$. **(7+7)**

5. a) Explain Gauss elimination method of solving the system of equations AX = B.

   b) Solve using Crout's method :

   $4x_1 + 2x_2 + 14x_3 = 14$

   $2x_1 + 17x_2 - 5x_3 = -101$

   $14x_1 - 5x_2 + 83x_3 = 155$. **(7+7)**

P.T.O.

6. a) Obtain the Gauss-Seidal iterative method in the form $X^{(K+1)} = HX^{(K)} + C$, $K = 0, 1, 2 \ldots$, where the quantities have their usual meaning.

   b) Outline Thomas algorithm for solving a tridiagonal system of equations and hence solve

   $$2.04\,x_1 - x_2 = 4.08$$
   $$-x_1 + 2.04x_2 - x_3 = 0.8$$
   $$-x_2 + 2.04\,x_3 - x_4 = 0.8$$
   $$-x_3 + 2.04x_4 = 200.8.$$

   (6+8)

7. a) Explain the method of cubic spline interpolation and derive the necessary equations to obtain the polynomials uniquely.

   b) Obtain a rational approximation $R_{2,2}(x)$ for e^x.

   (9+5)

8. a) Derive Gauss-Chebyshev three-point quadrature formula and hence evaluate

   $$\int_{-1}^{1} \sqrt{1-x^2}\,\cos(x)\,dx.$$

   b) Evaluate $\int_{2}^{3} \frac{2x}{1+x^4}\,dx$ using the Gauss-Legendre 2-point and 3-point quadrature rules. Compare with the exact solution.

   (8+6)